

Fractal Analysis

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A fractal

- a non-regular geometric shape
- can be split into parts
- which possesses self similarity

Naturally Occurring Fractal



A special type of broccoli, this cruciferous and tasty cousin of the cabbage is a particularly symmetrical fractal. Cook it for your favorite mathematician



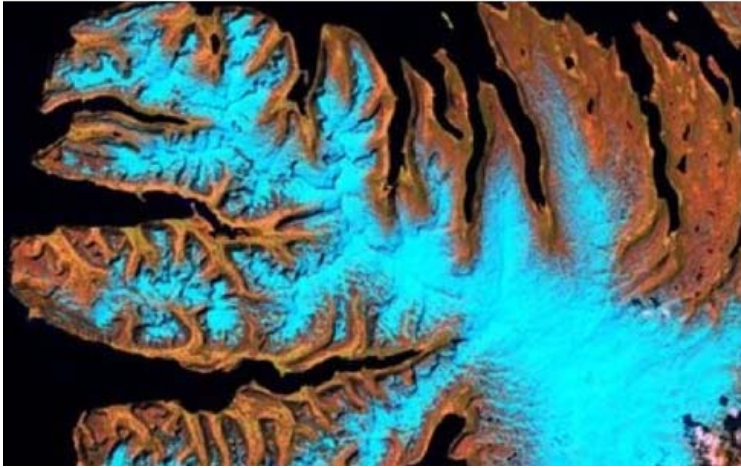
The fern is one of many flora that are fractal



The pineapple is an unusual fruit that is, in fact, a fractal .



Stalactites, Amazing fractal configurations



From the midwest of the United States to the icy fjords of Norway, fractals are often viewed by airline passengers



the macro
view of a leaf



Lightning's terrifying power is both awesome and beautiful. The fractals created by lightning are fascinatingly arbitrary and irregular



Sea shells -The nautilus is one of the most famous examples of a fractal in nature. The .perfect pattern is called a Fibonacci spiral



mountain ranges are considered loosely fractal



Snow flake



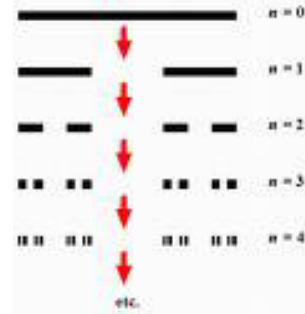
Sea urchins are compact, almost artistic little creatures; sea stars are more commonly referred to as starfish.

generating fractals

- possible to generate fractals
- using an analytical rule
- based upon a recursive mathematical relation.

A class of examples (for generating fractals)

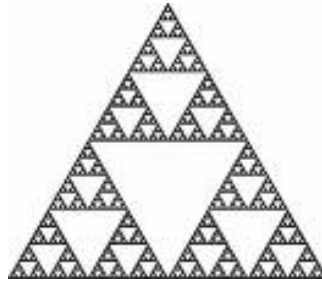
- Cantor set



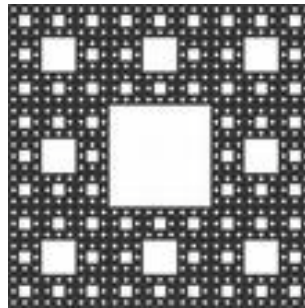
- Menger sponge



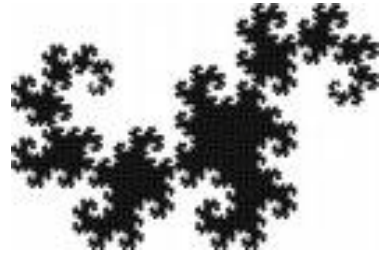
- Sierpinski triangle



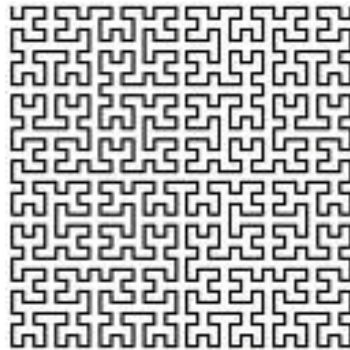
- Sierpinski carpet



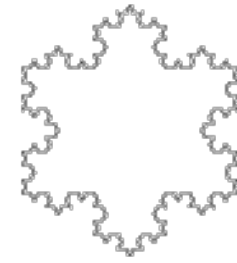
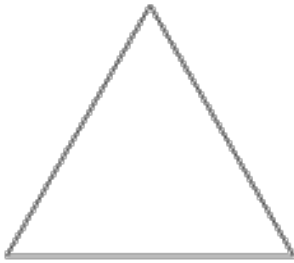
- Dragon curve



- Space filling curve

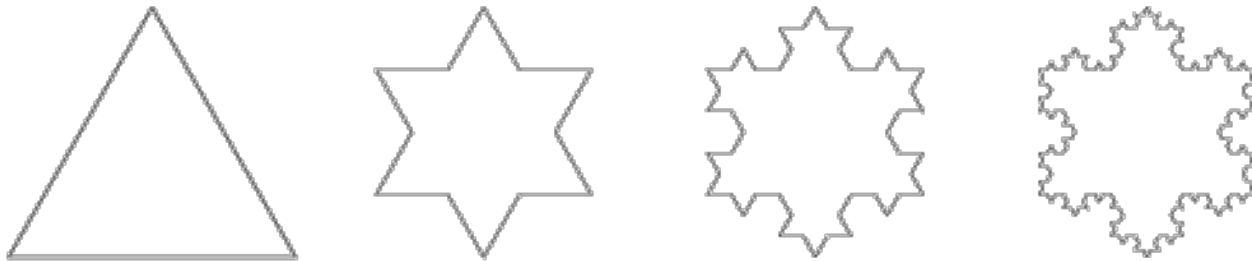


- Koch curve



Example (How to create Snowflake)

- is built by starting with an [equilateral triangle](#),
- removing the inner third of each side,
- building another [equilateral triangle](#) at the location where the side was removed,
- repeating the process indefinitely.



a very intuitive access to the language of fractal geometry

- consider a copy machine with an image reduction feature
- After some ten or so cycles any initial image would be reduced to just a point.



Mandelbrot is largely responsible for the present interest in Fractal Geometry

- Mandelbrot introduced the concept of fractal
- He showed how Fractals can occur in many different places in both Mathematics and elsewhere in Nature.

The most common dimensions in fractal analysis

- Hausdorff dimension
- Self similarity dimension
- Fractal dimension

Hausdorff dimension

- Fractal is an object whose Hausdorff dimension is greater than its topological dimension.
- the **Hausdorff dimension** is an extended non-negative real number associated to any metric space
- the Hausdorff dimension of a single point is zero,
- of a line is one,
- of the plane is two.
- many irregular sets (fractals) that have noninteger Hausdorff dimension

Examples of some fractals by their Hausdorff dimension

- Cantor set 0.6309
- Koch curve 1.2619
- Dragon curve boundary 1.5236
- Sierpinski triangle 1.585
- Sierpinski carpet 1.8928
- Menger sponge 2.7268

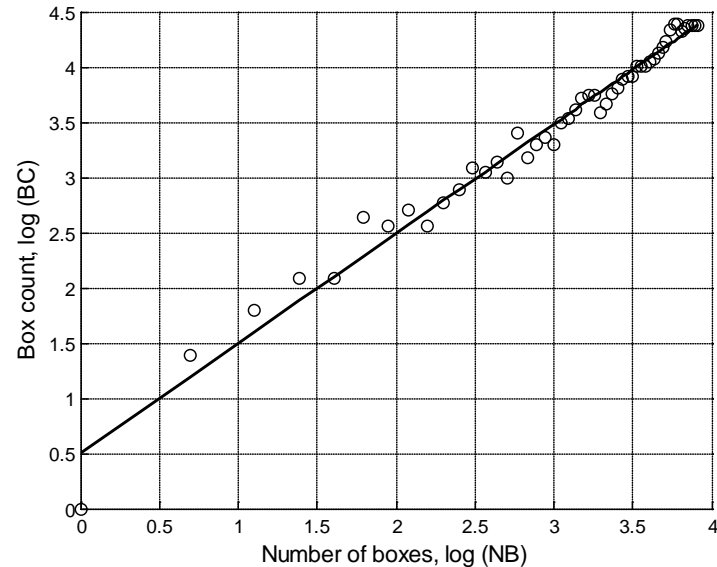
self similarity dimension

$$a \propto \frac{1}{S^D}$$

- D is the self similarity dimension
- a is the number of self similar pieces at reduction factor $(1/S)$, so

$$D = \frac{\log(a)}{\log\left(\frac{1}{S}\right)}$$

- D can be estimated by slope of the straight line approximation for a plot of $\log(a)$ vs. $\log(1/s)$.

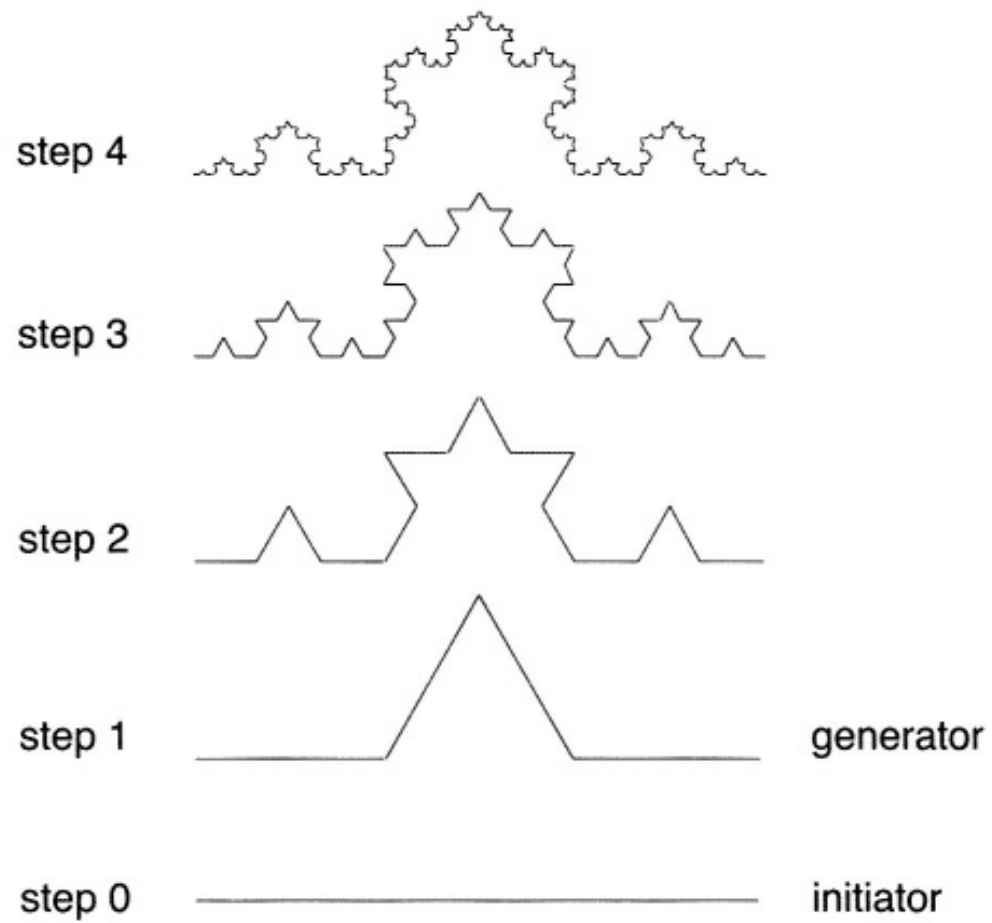


Fractal Dimension(FD)

- Fractal Dimension is a statistical quantity
- indicates how completely a fractal would fill the space in different scales or magnification

Example (How to compute FD of Koch Snowflake)

- At zeroth level ($n=0$) the step size is 1
- At the next level ($n=1$), the step size is $1/3$
- And so on



$$N_s = 4^n, L_s = 3^{-n} \Rightarrow n = -\frac{\log L_s}{\log 3} \Rightarrow N_s = 4^{-\log L_s / \log 3} \Rightarrow \log N_s = \frac{\log 4}{\log 3} \log L_s.$$

$$\frac{\log N_s}{\log L_s} = \frac{\log 4}{\log 3}$$

- FD= $\log 4 / \log 3$ which is approximately 1.26.

Two Methods for Estimating Fractal Dimension

- Box counting method(BCM)
- Ruler method(RM)

Illustrating BCM

- a grid of boxes is laid over the curve
- The number of grid boxes that contain a part of the curve is calculated.
- This is continued for increased numbers of squares,
- the FD is given by the gradient of the logarithm of the number of squares occupied by the edge contour, viz. $\log(a)$, over the logarithm of the number of squares which is $\log(1/s)$.

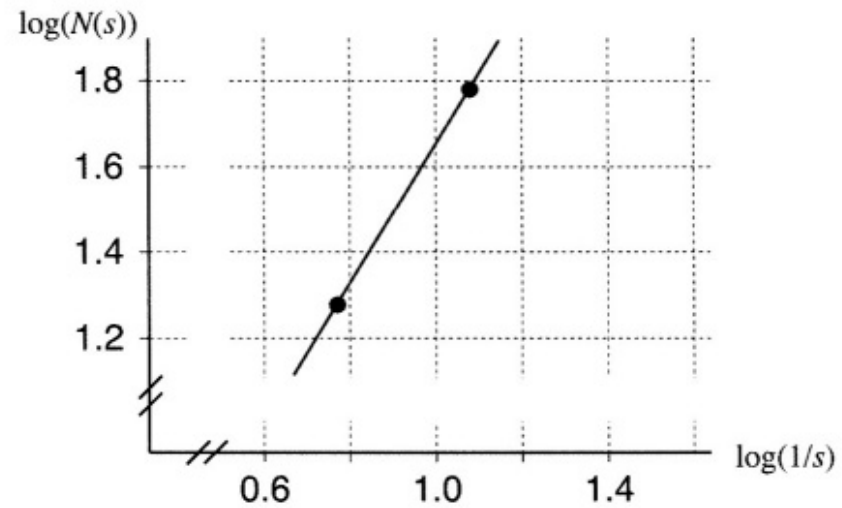
Example 1(BCM)



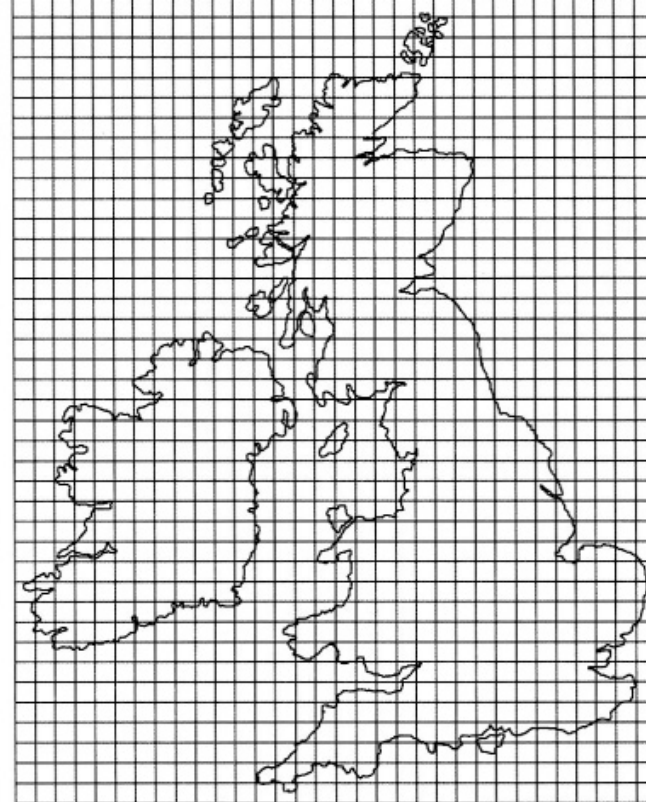
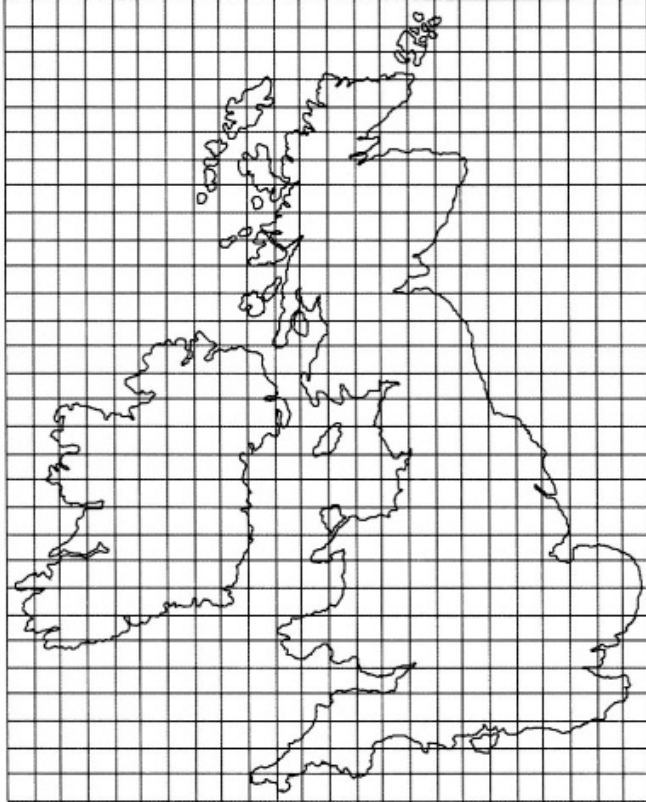
$s=1/6$ $N(s) = 19$



$s=1/12$ $N(s) = 52$



Example 2(BCM) (Coastline of Britain)



$$d = \frac{\log 283 - \log 194}{\log 32 - \log 24}$$

which is approximately 1.31

Illustrating RM

- change the length of the "ruler" that is used
- the FD is given by the gradient of the logarithm of the measured length over the logarithm of precision $1/s$.

Example of RM(Coastline of Britain)



Application of fractal analysis

- Medicine
- Chemistry
- Geology
- Economy
- Art
- Mathematics
- Ecology
- Oil and Gas Production
- Biology
- Human
- Surface
- Psychology
- food structure
- music
-

Application of Fractal analysis in Biomedical Images

- Abnormal regions in mammograms
- Irregularity of the borders of melanocytic lesions
- Cancerous vascular networks
- Magnetic Resonance Images of brain
- Ultrasonic images of liver
- Colonic cancer
- Vessel networks that surround a tumor and the hemodynamics within these vessel structures

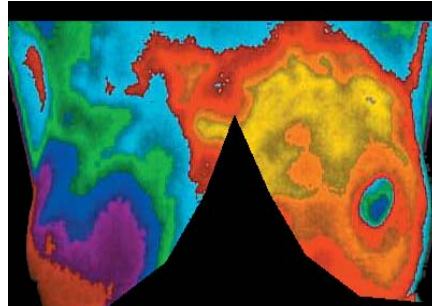
Fractal Analysis has potential to detect cancerous tumor

- Cancer is often characterized as a chaotic, poorly regulated growth.
- Cancerous cells, tumors, and vasculature defy have irregular shapes which can not be described by traditional Euclidean geometry based on smooth shapes such as line, plane, cylinder, and sphere.
- Fractal geometry is thus useful when the focus is on irregularities of tumor growth.

Estimating FD for breast IR images

1. We studied 15 breast thermograms (8 benign and 7 malignant cases)
2. The breast IR images are segmented by fuzzy c-means algorithm
3. The first hottest regions are identified based on the color related to the maximal temperature
4. The FD of The first hottest regions is calculated with BCMs

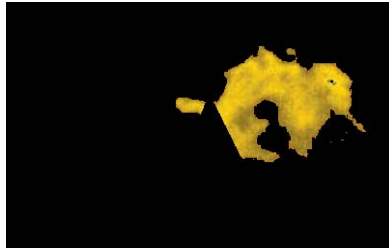
Example



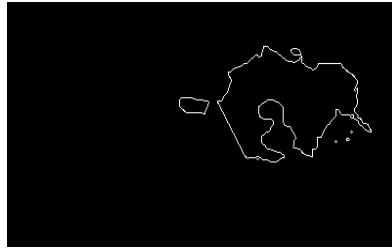
Malignant case



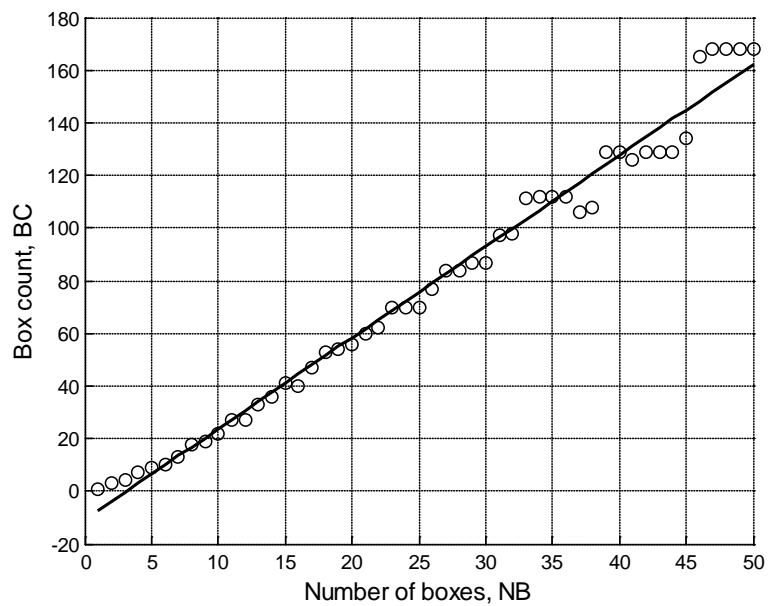
Segmentation of the image by fuzzy c-mean.



The first hottest regions.

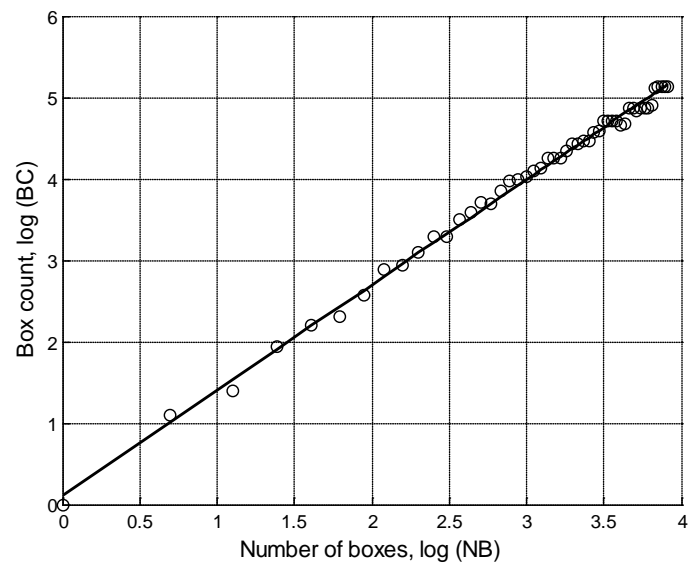


Boundary contours of the first hottest regions.



(1)

(1) Box count (BC) vs. number of boxes (NB).



(2)

(2) log (BC) vs. log (NB).

RESULTS

- The FDs for eight benign and seven malignant cases were calculated.
- The FD results for benign cases are not significantly different from 1, the topological dimension of a line.
- while those for malignant cases are significantly greater.

Calculated FDs for eight benign and seven malignant cases by BCM

(B: Benign, M: Malignant)

B1	0.9675	M1	1.3538
B2	.9911	M2	1.2992
B3	۱.۱321	M3	۱.۳479
B4	1.106	M4	1.2787
B5	1.0382	M5	1.4001
B6	0.9781	M6	1.331
B7	1.1218	M7	۱.۲۸۸۴
B8	1.0081		